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Utilization of Quartiles, Deciles & Arithmetic Mean as Auxiliary Variable for Precise Estimation of Population Variance

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ABSTRACT

In this manuscript, an improved estimator for the estimation of population variance is proposed by incorporating the linear combination of Quartiles arithmetic mean, and deciles arithmetic means as auxiliary information. The bias and mean square error are derived up to their first order of approximation. An empirical study demonstrates the performance and justifies the usefulness of the proposed estimators.

Key words: *Quartile arithmetic mean; Deciles arithmetic mean; Bias; Mean square error; Efficiency.*

INTRODUCTION

The proper use of auxiliary variable plays an eminent and solicitor role in developing the efficient estimators", when there exists a close association between auxiliary and study variables. Different authors have addressed the technique of searching efficient estimators in the literature such as Isaki[1] where the author has utilized standard deviation as auxiliary information to enhance the efficiency of the estimator for the estimation of population variance. Upadhyaya and Singh[2] incorporated the coefficient of kurtosis as an auxiliary variable to increase the efficiency of an estimator for the estimation of population variance. Kadilar and Cingi[3] "introduced the coefficient of variation as an auxiliary variable to improve the efficiency of the estimator. In the same line of thought, authors such as Sarandal CE[4], Bhat *et al.*[5] have also utilized this auxiliary information to enhance the efficiency of estimators. The Strategy of modifying estimators, by using auxiliary information is now regularly carried out in the field of survey sampling to improve the efficiency of estimators to obtain precise and reliable estimates in survey estimation.

Let us consider a finite population is having N distinct and identifiable units. Let " $Y_i(i=1,2,3,,,,,,,N)$, denote the observations on study variable Y and " $X_i(i=1,2,3....N)$, denote the observations on an auxiliary variable X , where the information about the auxiliary variable is known." In this Study, our aim is to estimate the finite population variance by introducing new and improved estimators in sample surveys".

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MATERIAL AND METHODS

Notations:

$N = population-size, n = Sample-size, \gamma = \frac{1}{n}, Y = study-\text{var}iable, X = Auxiliary-\text{var}iable,$

$\overline{X}$ & $\overline{Y} = Population-means, \bar{x}$ & $\bar{y} = Sample-means, S_y^2$ & $S_x^2 = Population-\text{var}iances,$

C_x & $C_y = Cofficient-of-\text{var}iations, \rho = correlation-cofficient, \beta_{1x} = Sekewness$ & $\beta_{2x} = Kurtosis$

$Q_1 = Quartile-I, Q_2 = Quartile-II, Q_3 = Quartile-III,$

$Q_d = Quartile-Deviation, Q_r = Quartile-Range, Q_a = Quartile-Average, D = Decile,$

$Q_{A.M} = Quartile-airthmetic-mean, D_{A.M} = Decile-airthmetic-mean,$

$B(.) = Bias$ & $MSE(.) = Mean-square-error$

PROPOSED ESTIMATOR

$$\hat{S}_M^2 = s_y^2\left[\frac{S_x^2 + \left(Q_{A.M} + D_{A.M}\right)\frac{\beta_{2x}}{\beta_{1x}}}{s_x^2 + \left(Q_{A.M} + D_{A.M}\right)\frac{\beta_{2x}}{\beta_{1x}}}\right] \tag{3.1}$$

The above estimator is based on the linear combination of quartiles, deciles and arithmetic mean of the auxiliary variable in order to get the best precise estimators which have less bias and mean square error as compared to the already existing estimators in the literature.

We have derived the bias and mean square error of proposed estimator up to the first order of approximation as given below

Let $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$. Further, we can write $s_y^2 = S_y^2(1+e_0)$ $s_x^2 = S_x^2(1+e_1)$ and from the definition of e_0 and e_1 we obtain:

$E[e_0] = E[e_1] = 0$, $E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)} - 1)$, $E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)} - 1)$, $E[e_0 e_1] = \frac{1-f}{n}(\lambda_{22} - 1)$

The proposed estimator is given as:

$$\hat{S}_M^2 = s_y^2\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + \alpha a_i}\right] \tag{4.1}$$

$$\Rightarrow \quad \hat{S}_M^2 = s_y^2(1+e_0)\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + e_1 S_x^2 + \alpha a_i}\right]$$

$$\Rightarrow \quad \hat{S}_M^2 = \frac{S_y^2(1+e_0)}{(1+A_M e_1)} \text{ Where } A_M = \frac{S_x^2}{S_x^2 + \alpha a_i}$$

$$a_i = (Q_{A.M} + D_{A.M})\left(\frac{\beta_{2x}}{\beta_{1x}}\right) and \alpha = 1$$

$$\Rightarrow \quad \hat{S}_M^2 = S_y^2(1+e_0)(1+A_M e_1)^{-1} \tag{4.2}$$

$$\Rightarrow \quad \hat{S}_M^2 = S_y^2(1+e_0)(1-A_M e_1 + A_M^2 e_1^2 - A_M^3 e_1^3 + ...) \tag{4.3}$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}_M^2 = S_y^2 + S_y^2 e_0 - S_y^2 A_M e_1 - S_y^2 A_M e_0 e_1 + S_y^2 A_M^2 e_1^2 \tag{4.4}$$

$$\Rightarrow \quad \hat{S}_M^2 - S_y^2 = S_y^2 e_0 - S_y^2 A_M e_1 - S_y^2 A_M e_0 e_1 + S_y^2 A_M^2 e_1^2 \tag{4.5}$$

By taking expectation on both sides of (4.6), we get

$$E(\hat{S}_M^2 - S_y^2) = S_y^2 E(e_0) - S_y^2 A_M E(e_1) - S_y^2 A_M E(e_0 e_1) + S_y^2 A_M^2 E(e_1^2) \tag{4.6}$$

$$Bias(\hat{S}_M^2) = S_y^2 A_M^2 E(e_1^2) - S_y^2 A_M E(e_0 e_1) \tag{4.7}$$

$$Bias\left(\hat{S}_M^2\right) = \gamma S_y^2 A_M \left[A_M\left(\beta_{2x} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{4.8}$$

Squaring both sides of (4.6) and neglecting the terms more than 2nd order and taking expectation, we get

$$E\left(\hat{S}_M^2 - S_y\right)^2 = \gamma S_y^4 E\left(e_0^2\right) + S_y^4 A_M^2 E(e_1^2) - 2S_y^4 A_M E\left(e_0 e_1\right)$$

$$MSE\left(\hat{S}_M^2\right) = \gamma S_y^4\left[(\beta_{2y} - 1) + A_M^2\left(\beta_{2x} - 1\right) - 2A_M\left(\lambda_{22} - 1\right)\right]$$

EFFICIENCY CONDITIONS

We have derived the efficiency conditions of proposed estimators with other existing estimators under which proposed estimators are performing better than the existing estimators. The bias and Mean square error of existing ratio type estimators up to the first order of approximation is given by

$$Bias\left(\hat{S}_K^2\right) = \gamma S_y^2 R_K\left[R_K\left(\beta_{2x} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{5.1}$$

$$MSE\left(\hat{S}_K^2\right) = \gamma S_y^4\left[\left(\beta_{2y} - 1\right) + R_K^2\left(\beta_{2x} - 1\right) - 2R_K\left(\lambda_{22} - 1\right)\right] \tag{5.2}$$

$R_K = Existing.cons\tan t$

$, K = 1,2,3,4........$

Bias, MSE and constant of proposed estimators is given by

$$Bias\left(\hat{S}_P^2\right) = \gamma S_y^2 R_P\left[R_P\left(\beta_{2x} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{5.3}$$

$$MSE\left(\hat{S}_P^2\right) = \gamma S_y^4\left[\left(\beta_{2y} - 1\right) + R_P^2\left(\beta_{2x} - 1\right) - 2R_P\left(\lambda_{22} - 1\right)\right] \tag{5.4}$$

$R_P = proposed.cons\tan t$

$P = 1,2,3.............$

From Equation (5.2) and (5.4), we have

$$MSE\left(\hat{S}_P^2\right) \le MSE\left(\hat{S}_K^2\right) if \lambda_{22} \ge 1 + \frac{\left(R_P + R_K\right)\left(\beta_{2x} - 1\right)}{2}$$

$$MSE\left(\hat{S}_P^2\right) \le MSE\left(\hat{S}_K^2\right)$$

$$\gamma S_y^4\left[\left(\beta_{2y} - 1\right) + R_P^2\left(\beta_{2x} - 1\right) - 2R_P\left(\lambda_{22} - 1\right)\right] \le \gamma S_y^4\left[\left(\beta_{2y} - 1\right) + R_K^2\left(\beta_{2x} - 1\right) - 2R_K\left(\lambda_{22} - 1\right)\right] \tag{5.5}$$

$$\Rightarrow \left[\left(\beta_{2y} - 1\right) + R_P^2\left(\beta_{2x} - 1\right) - 2R_P\left(\lambda_{22} - 1\right)\right] \le \left[\left(\beta_{2y} - 1\right) + R_K^2\left(\beta_{2x} - 1\right) - 2R_K\left(\lambda_{22} - 1\right)\right] \tag{5.6}$$

$$\Rightarrow \left[1 + R_P^2\left(\beta_{2x} - 1\right) - 2R_P\left(\lambda_{22} - 1\right)\right] \le \left[1 + R_K^2\left(\beta_{2x} - 1\right) - 2R_K\left(\lambda_{22} - 1\right)\right] \tag{5.7}$$

$$\Rightarrow \left(\beta_{2x} - 1\right)\left(R_P^2 - R_K^2\right)\left[-2R_P\left(\lambda_{22} - 1\right)\right] \le \left[-2R_K\left(\lambda_{22} - 1\right)\right] \tag{5.8}$$

$$\Rightarrow \left(\beta_{2x} - 1\right)\left(R_P^2 - R_K^2\right)\left[-2\left(\lambda_{22} - 1\right)\left(R_P - R_K\right)\right] \le 0 \tag{5.9}$$

$$\Rightarrow (\beta_{2x}-1)(R_P^2-R_K^2) \le [2(\lambda_{22}-1)(R_P-R_K)] \quad (5.10)$$

$$\Rightarrow (\beta_{2x}-1) \le \frac{2(\lambda_{22}-1)(R_P-R_K)}{(R_P^2-R_K^2)} \quad (5.11)$$

$$\Rightarrow (\beta_{2x}-1) \le \frac{2(\lambda_{22}-1)(R_P-R_K)}{(R_P-R_K)(R_P+R_K)} \quad (5.12)$$

$$\Rightarrow (\beta_{2x}-1)\ (R_P+R_K) \le 2(\lambda_{22}-1) \quad (5.13)$$

By solving equation (5.13), we get

$$MSE(\hat{S}_P^2) \le MSE(\hat{S}_K^2)\ if\ \lambda_{22} \ge 1+\frac{(R_P+R_K)(\beta_{2x}-1)}{2}$$

NUMERICAL ILLUSTRATION

We use the data set presented in Sarandal *et al.*[4] concerning (P85) 1985 population considered as Y and RMT85 Revenue from 1985 municipal taxation in millions of kronor considered as X.

$$N=234, n=35, \bar{Y}=29.3626, \bar{X}=245.088, S_y=51.556, S_x=596.332, \rho=0.96, \beta_{2y}=89.231, \beta_{2x}=89.189$$
$$\lambda_{22}=4.041, \beta_{1x}=8.83, \beta_{1y}=8.27, TM=167.4, Q_1=67.75, Q_2=113.5, Q_3=230.25, C_x=2.43, D_1=49.0,$$
$$D_2=63.0, D_3=75.0, D_4=90.0, D_5=113.5, D_6=145.9, D_7=197.9, D_8=271.1, D_9=467.5, D_{10}=6720.0$$

Table 1: Bias and MSE of existing and proposed estimators

Estimators	Bias	MSE
Existing estimators		
Isaki [1]	5494.93	29216819.02
Upadhyaya &Singh [2]	5483.00	29187658.80
Kadilar & Cingi [3]	5483.00	29187658.80
Proposed estimator M[4]	**3449.81**	**23889076.41**

CONCLUSION

Above study clearly reveals that the proposed estimator has shown better performance in terms of bias and mean square error as compared to other existing estimators. Hence the proposed estimator has better applications and is recommended for future studies.

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